METHOD OF RELIABILITY-BASED SEISMIC DESIGN.
I: EQUIVALENT NONLINEAR SYSTEMS

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ABSTRACT: This is the first part of a two-part investigation on the method of reliability-based seismic design. In reliability analysis and reliability-based design under stochastic loads such as seismic excitations, repeated time-history solutions of multi-degree-of-freedom (MDOF) inelastic structures are often required, which can become computationally expensive. To alleviate this difficulty, an approximate method is developed by replacing the MDOF system with a simple equivalent nonlinear system (ENS) that retains the dynamic characteristics of the first two modes and the global yielding behavior of the system. The equivalence is achieved by the use of two empirical response modification factors based on regression analyses of responses to a large number of actual ground accelerations recorded in past earthquakes. Numerical examples on reliability analysis of the special moment resistant space frame (SMRSF) under seismic excitations are given. The computational advantage and accuracy of the method are demonstrated.

INTRODUCTION

In reliability evaluation of buildings and structures under seismic excitation, repeated solutions of the response of multi-degree-of-freedom (MDOF) systems in the inelastic range are often required and the computation can become excessive. The problem becomes more serious in developing reliability-based design for a building population since in search for the optimal design parameters, evaluation of the reliability under seismic and other loads is necessary for various combinations of building type (e.g., story height and structural frame system) and design parameters (e.g., load factors and drift limits), and the number of such combinations can become very large. The computational effort can be reduced by using approximate equivalent, single degree-of-freedom (SDOF) linear elastic systems with some response modification factors such that the responses of the equivalent systems match those of the original systems. For example, Inoue and Cornell (1991) have proposed a method in which a MDOF response factor in conjunction with a nonlinear spectral reduction factor were used to obtain the equivalent response of a linear SDOF system. Procedures of spectral reduction factors for inelastic systems have also been developed for researchers for design under seismic loads [e.g., Riddell et al. (1979); Kennedy et al. (1984); Nassar and Krawinkler (1991)]. While such methods provide a simple and convenient means for analysis and design of MDOF systems, the accuracy is inherently limited because of the basic differences between the dynamics of an inelastic MDOF system and a linear elastic SDOF system. For example, Fig. 1 shows the mean and standard deviation of the local spectral reduction factor (Inoue and Cornell 1991) as a function of ductility for a two-story special moment resistant space frame (SMRSF) using nine historical earthquake records. Fig. 2 shows the deviation of local drift (0.5-3.0% of story height) limit-state probabilities of the two-story building modeled by the equivalent linear SDOF system from those based on nonlinear analysis of the original MDOF system. Seventy simulated ground motions for two sites in California (Wen et al. 1994) were used to calculate the limit-state probabilities. The large variability in the factors for the equivalent system and the significant deviation and bias in the limit-state probability estimates indicate that using these factors in developing a reliability-based design procedure may not give satisfactory results.

The purpose of the present study is to develop an alternative procedure of approximating the original MDOF systems, which may improve the accuracy while keeping the computation within a reasonable limit. Such a method is required in a reliability-based calibration of design parameters under seismic loads as shown in the companion paper (Han and Wen 1997). The idea is to replace the original structure by a simple equivalent nonlinear system (ENS) that retains the most important properties of the original system, i.e., the dynamic characteristics of the first two modes and the local and global yielding behavior of the MDOF system. The system response is described by the maximum global (building) and local (interstory) drifts. The equivalency is achieved by the use of two empirical modification factors, a global response factor and a local response factor, applied to the responses of ENS to match those of the original MDOF system. The inclusion of the second mode would improve the accuracy of the equivalent systems at a modest increase of computational effort. These response modification factors are obtained by extensive regression analysis based on results of responses of the MDOF system and ENS to actual ground accelerations recorded in past earthquakes. The method is described in the following. Only SMRSF steel structures are studied, although the methodology can be applied to other types of structure. More details can be found in Han and Wen (1994).

FIG. 1. Local Spectral Reduction Factor as Function of Ductility for Two-Story SMRSF

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ENS is defined as a system consisting of two SDOF systems whose dynamic properties (natural frequency, mode shape, and modal participation factor) are the same as those of the first two modes of the original system. It is assumed that both SDOF systems have a yield displacement equal to the global yield displacement associated with the MDOF structure. The global yield displacement is determined by a static pushover analysis in which a linear vertical distribution of the lateral force is assumed and applied proportionally on the building, well into inelastic range. The displacement at the top of the building is monitored, and a force-displacement relationship is obtained that characterizes the inelastic response behavior of the MDOF system. The well known DRAIN-2DX (Allahabadi and Powell 1988) is used in the pushover analysis. Since for MDOF systems the yielding of the system occurs incrementally, the point of intersection of the preyielding and postyielding slopes in the force-deflection curve is taken as the yield point and based on which the global yield displacement is determined. Fig. 3 shows an example of such an analysis for a two-story steel special moment frame designed in accordance with the Uniform Building Code (UBC) (1991). A bilinear restoring force model is then used for the inelastic response analysis of the two SDOF systems under the excitation of ground acceleration. Such a model has been commonly used for steel frames (Wakabayashi 1986). Again the displacement at the top of the building is monitored. The analysis is otherwise the same as a modal response analysis, and the displacements of the two SDOF systems are then combined by modal superposition. It is fully recognized that modal analysis is strictly valid only for linear systems. For many structures, however, it has been observed that deviation from linear behavior may not be severe, and one can still take advantage of the modal analysis to obtain an approximate solution. To obtain the maximum overall (top) response and the interstory drift of the original system for reliability analysis, the responses of the equivalent system are multiplied by a global and a local response modification factor, $R_g$ and $R_L$, respectively. Values $R_g$ and $R_L$ are determined from regression analysis of the responses of the two systems under historical earthquakes. The procedure is outlined in the following.

FIG. 2. Histogram of Deviation of Local Drift Limit-State Probabilities of Equivalent Linear SDOF System from Those of Nonlinear MDOF System

DETERMINATION OF RESPONSE MODIFICATION FACTORS

Selection and Design of Representative Structures

This study concentrates on low to midrise structures. Seven office buildings from one to 12 stories and one to four bays are selected. The structures are assumed to be located in zone 4 of UBC. The design seismic force is determined in accordance with the provisions of UBC (1991). Only SMRSFs are considered. The frames are designed according to UBC (1991) and the American Institute of Steel Construction (AISC) (1989) using a computer program IGRESS-2 (1989). The dimensions of the seven structures are shown in Fig. 4. Only one high-rise building is considered [more than nine stories, according to Rojahn and Sharpe (1985)]. The important dynamic properties of the structures are shown in Table 1. In the dynamic analysis of these structures, a 5% damping ratio is used, which is consistent with the values in current seismic provisions for buildings. In the ENS analyses, a strain hardening ratio of 10% at the global (system) level is assumed, which may be on the conservative side for most structures (Osteraas and Krawinkler 1990). The moment frames are assumed to behave as a ductile system under seismic loads.

Selection of Historical Earthquake Records

A suite of 88 real earthquake records are used in the dynamic-response time-history response analyses for the calibration of $R_g$ and $R_L$. The records consist of moderate to severe ground-acceleration time histories in North America since 1933, including those of the recent North Ridge earthquake. Of these, 66 were obtained from the U.S. Geological Survey digital data series including seven 1971 San Fernando records and six 1979 Imperial Valley records (Seeikins et al. 1992). Earthquakes in Japan, Chile, Managua, and Mexico are also considered. These are uncorrected accelerograms. The computer program BAP (Converse 1992) is used for corrections in which procedures similar to those outlined in Naeim and Anderson (1993). The magnitude of the earthquakes range from 4.4 to 8.1, peak ground acceleration from 0.03 to 1.17 g, and the source distance from 0 to 400 km. Fig. 5 shows the plot of magnitude and distance of these earthquakes. The peak ground accelerations versus the distances of these earthquakes are also shown. The characteristics of the excitations considered, therefore, cover a wide range for the regression analysis.

Regression Analysis of $R_g$ and $R_L$

For each accelerogram the dynamic response of the original MDOF system for each of the seven structural systems is calculated using DRAIN-2DX. A parallel dynamic analysis of...
ENS under the same ground acceleration is carried out. The
global response modification factor, $R_g$, is defined as the ratio
of the maximum displacement at the top of ENS ($G_0$) to that
of the original MDOF structure ($G_0$) as follows:

$$R_g = \frac{G_0}{G_m}$$  \hfill (1)

The local response modification factor, $R_L$, is defined as the
ratio of the maximum global ductility to the local (interstory
drift) ductility of the original MDOF structure as follows:

$$R_L = \frac{G_{UL} / Y}{\max(d'/y')},$$  \hfill (2)

where $d'$ = interstory drift at the $i$th story; $y'$ = $i$th story yield

$\gamma = \text{global yield displacement (cm)}$; and $H = \text{building height (m)}$. 

It is assumed that the response modification factors can be
modeled by polynomial functions of the global ductility factor
($\mu_g$), which is defined as the ratio of maximum roof displacement ($u_r$) to global yield displacement ($u_y$) and the modal par-
The global response modification factor is expressed as a function of global ductility factor $\mu$ as follows:

$$ R_G = C_0(\gamma) + C_1(\gamma)\mu + C_2(\gamma)\mu^2 $$  \hspace{1cm} (3)

where $C_0$, $C_1$, and $C_2$ are assumed to be linear functions of the modal participation factor ratio $\gamma$ defined as

$$ \gamma = \frac{|P_1| + |P_2|}{\sum |P_i|} $$  \hspace{1cm} (4)

where $P_i$ is the $i$th modal participation factor; and $n$ is the number of modes considered. To determine the preceding coefficients, values of $R_G$ for the previous seven buildings under the excitation of all 88 acceleration records are first calculated using the program DRAIN-2DX (Allahabadi and Powell 1988). A two-stage regression analysis is then performed. In the first stage, coefficient $C$'s are determined for each of the seven structures with a specified $\gamma$ value, and in the second stage, a regression analysis of the coefficients on $\gamma$ is carried out. The results of the two-stage regression analysis are

$$ C_0 = 0.9695 + 0.0178\gamma; \quad C_1 = -0.1664 + 0.2016\gamma $$  \hspace{1cm} (5, 6)

$$ C_2 = 0.1473 - 0.1467\gamma $$  \hspace{1cm} (7)

Fig. 6 shows the comparison of the data points with the regression results given by (3)-(7). It is seen that for $\mu$ less than 0.5, the variability of $R_G$ is small. For $\mu$ greater than 0.5, the coefficient of variation of $R_G$ for the one-, two-, five-, nine-, and 12-story structures is 5, 6, 10, 11, and 10%, respectively. As an approximation, a constant coefficient of variation of 10% is assumed in the evaluation of the limit-state probability using the response modification factors.

A similar two-stage regression analysis is carried out to calibrate the local response modification factor $R_L$. It is found that the variation of $R_L$ with $\mu$ is small and can be neglected. Value $R_L$ is hence given by linear function of $\gamma$ only as follows:

$$ R_L = 0.3627 + 0.4774\gamma $$  \hspace{1cm} (8)

Fig. 7 shows the comparison of the data points with the regression equation of $R_L$. Fig. 8 shows the regression equation of $R_L$ as a function of $\gamma$.

The two coefficients are applied to the response of the equivalent nonlinear system. They function as correction factors dependent primarily on the inelastic response severity. No significant dependence on the seismicity parameters such as earthquake magnitude and distance has been found. Similar conclusions have been reached by previous studies [e.g., Inoue and Cornell (1991)].
where the following is assumed:

\[ \frac{\sqrt{y^2}}{Y} = \frac{h'}{H} \]  

Eq. (13) is an approximate relationship that simplifies the calculation since the story yield and story height need not be considered. The advantage of formulation through (11) and (12) is that one needs to do large number of response time-history analyses of ENS only. The effect of the small scatter of \( R_G \) and \( R_L \) can be incorporated into the previous limit-state probability evaluation by modeling these two response modification factors as normal random variables with the mean values and coefficients of variation according to the results of the foregoing regression analyses. The effect, however, is generally small since the uncertainty in the seismic excitation is much larger and is the dominant factor in the limit-state probability evaluation.

**NUMERICAL RESULTS**

The accuracy of the proposed method is verified by comparison of the limit-state probabilities with those for the original nonlinear MDOF systems, hereafter referred to as NMS. The seven steel SMRSFs in the foregoing are used for this purpose. The sites chosen are downtown Los Angeles (Santa Monica Boulevard) and Imperial Valley as in the previous study of reliability evaluation of steel buildings (Wen et al. 1994). The Santa Monica Boulevard site is 60 km from the Mojave segment of the Southern San Andrea fault, and the Imperial Valley site is 5 km from the Imperial fault. Future earthquakes are modeled either as characteristic or noncharacteristic events. The former have relative regular recurrence periods, and a narrow range of magnitudes are therefore modeled by a renewal process of fixed magnitude. The local, noncharacteristic events are modeled by Poisson process. Besides occurrence time and magnitude, the major parameters of the characteristic event are epicentral distance to the site and intensity attenuation, whereas for noncharacteristic events, the major parameters considered are the local intensity (Modified Mercalli Intensity) and duration. Using these parameters, sample time histories of ground acceleration of future events at the site are simulated as nonstationary random processes with frequency and intensity modulation (Yeh and Wen 1989; Eliopoulos and Wen 1991). Details of the parameters of the simulated ground motions are given in Wen et al. (1994). The response time histories of both the nonlinear MDOF system and ENS are then calculated. The program DRAIN-2 DX is again used for analysis of a nonlinear MDOF system. The limit-state probabilities in terms of drift levels being exceeded in the 50-year time window after 1995 are presented using ENS as indicated in (11)–(13) and are compared with those using the original nonlinear MDOF systems. The two
random variables \( R_0 \) and \( R_L \) are generated on computer and used in the previous limit-state probability evaluation.

For the Los Angeles site, since both types of event may contribute to the limit-state probability, 50 noncharacteristic events and 10 characteristic events are generated, hence 70 response time histories are calculated and from which the limit-state probabilities are evaluated. The conditional probabilities of response given the occurrence of an event are first evaluated by using extreme value distributions that fit the results of time-history response analysis. These probabilities are then combined with the occurrence probabilities of the characteristic and noncharacteristic events to arrive at the 50-year limit-state probabilities assuming that the occurrences of these two types of event are independent. Tables 2 and 3 show the 50-year global and local limit-state probabilities for both ENs and nonlinear MDOF systems of the seven SMRSPs. The drift limit covers a range of 0.5-3% of building or story height, which corresponds to levels from serviceability to ultimate limit. For a steel frame building, at 3% drift level it will probably suffer severe nonstructural or even structural damages. The probabilities are also compared in Fig. 9. Since the probabilities of different limit states of the representative structures cover a wide range and several orders of magnitude, the probabilities are shown in log scale. It is seen that the ENs method gives good prediction of the limit-state probabilities. The computation required is only a small fraction of that for the original MDOF systems. For the Imperial Valley site, 50 characteristic events are generated. The important near-source directivity effect is also considered by assuming that the rupture may propagate in two opposite directions along the Imperial Fault as was observed in the 1940 El Centro earthquake and the 1979 Imperial Valley earthquake. Details can be found in Eliopoulos and Wen (1991). The procedure for calculating the 50-year limit-state probabilities is the same as for the Los Angeles site. Tables 4 and 5 show the limit-state probabilities. The probabilities based on ENs and the original MDOF system are also compared in Fig. 10. It is seen that the agreements are still good though the scatters are larger. It may be attributed to the larger inelastic response at the Imperial Valley site and the fact that scatter in the two response modification factors \( R_0 \) and \( R_L \) increase with inelastic deformation of the structure. Finally, Fig. 11 shows the histogram of the deviation of the local drift probabilities of a two-story building from those based on analyses of the original MDOF systems. Note that compared with the results shown in Fig. 2, the ENs method gives much less scatter and bias than the method using an equivalent linear SDOF system.

### TABLE 2. Global Limit-State Probabilities (Los Angeles Site)

<table>
<thead>
<tr>
<th>Structure number</th>
<th>System (1)</th>
<th>0.5% of height (2)</th>
<th>1.0% of height (3)</th>
<th>1.5% of height (4)</th>
<th>2.0% of height (5)</th>
<th>2.5% of height (6)</th>
<th>3.0% of height (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NMS</td>
<td>0.2415</td>
<td>0.0749</td>
<td>0.0387</td>
<td>0.0241</td>
<td>0.0167</td>
<td>0.0123</td>
</tr>
<tr>
<td>2</td>
<td>ENs</td>
<td>0.2443</td>
<td>0.0741</td>
<td>0.0383</td>
<td>0.0237</td>
<td>0.0163</td>
<td>0.0120</td>
</tr>
<tr>
<td>3</td>
<td>NMS</td>
<td>0.3021</td>
<td>0.0906</td>
<td>0.0476</td>
<td>0.0300</td>
<td>0.0210</td>
<td>0.0156</td>
</tr>
<tr>
<td>4</td>
<td>ENs</td>
<td>0.3050</td>
<td>0.0898</td>
<td>0.0468</td>
<td>0.0294</td>
<td>0.0205</td>
<td>0.0152</td>
</tr>
<tr>
<td>5</td>
<td>NMS</td>
<td>0.3566</td>
<td>0.0692</td>
<td>0.0361</td>
<td>0.0227</td>
<td>0.0158</td>
<td>0.0112</td>
</tr>
<tr>
<td>6</td>
<td>ENs</td>
<td>0.3556</td>
<td>0.0695</td>
<td>0.0364</td>
<td>0.0230</td>
<td>0.0161</td>
<td>0.0120</td>
</tr>
<tr>
<td>7</td>
<td>NMS</td>
<td>0.3343</td>
<td>0.0665</td>
<td>0.0356</td>
<td>0.0239</td>
<td>0.0162</td>
<td>0.0121</td>
</tr>
<tr>
<td>8</td>
<td>ENs</td>
<td>0.3253</td>
<td>0.0669</td>
<td>0.0358</td>
<td>0.0240</td>
<td>0.0163</td>
<td>0.0121</td>
</tr>
</tbody>
</table>

Note: NMS is the nonlinear MDOF system. ENs is the equivalent nonlinear system.

### FIG. 9. Comparison of Failure Probabilities of ENs and Nonlinear MDOF Systems at Los Angeles Site: (a) Global Limit State; (b) Local Limit State

### TABLE 4. Global Limit-State Probabilities (Imperial Valley Site)

<table>
<thead>
<tr>
<th>Structure number</th>
<th>System (1)</th>
<th>0.5% of height (2)</th>
<th>1.0% of height (3)</th>
<th>1.5% of height (4)</th>
<th>2.0% of height (5)</th>
<th>2.5% of height (6)</th>
<th>3.0% of height (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NMS</td>
<td>0.8264</td>
<td>0.7045</td>
<td>0.2956</td>
<td>0.1155</td>
<td>0.0383</td>
<td>0.0109</td>
</tr>
<tr>
<td>2</td>
<td>ENs</td>
<td>0.8264</td>
<td>0.6902</td>
<td>0.2879</td>
<td>0.0916</td>
<td>0.0245</td>
<td>0.0062</td>
</tr>
<tr>
<td>3</td>
<td>NMS</td>
<td>0.8258</td>
<td>0.5519</td>
<td>0.1930</td>
<td>0.0589</td>
<td>0.0166</td>
<td>0.0045</td>
</tr>
<tr>
<td>4</td>
<td>ENs</td>
<td>0.8230</td>
<td>0.5595</td>
<td>0.2154</td>
<td>0.0568</td>
<td>0.0140</td>
<td>0.0041</td>
</tr>
<tr>
<td>5</td>
<td>NMS</td>
<td>0.8253</td>
<td>0.4241</td>
<td>0.0886</td>
<td>0.0147</td>
<td>0.0023</td>
<td>0.0003</td>
</tr>
<tr>
<td>6</td>
<td>ENs</td>
<td>0.7942</td>
<td>0.4551</td>
<td>0.1020</td>
<td>0.0154</td>
<td>0.0021</td>
<td>0.0003</td>
</tr>
<tr>
<td>7</td>
<td>NMS</td>
<td>0.8252</td>
<td>0.4607</td>
<td>0.1090</td>
<td>0.0255</td>
<td>0.0051</td>
<td>0.0001</td>
</tr>
<tr>
<td>8</td>
<td>ENs</td>
<td>0.7972</td>
<td>0.4495</td>
<td>0.1023</td>
<td>0.0160</td>
<td>0.0023</td>
<td>0.0003</td>
</tr>
<tr>
<td>9</td>
<td>NMS</td>
<td>0.8247</td>
<td>0.4282</td>
<td>0.0923</td>
<td>0.0157</td>
<td>0.0026</td>
<td>0.0004</td>
</tr>
<tr>
<td>10</td>
<td>ENs</td>
<td>0.8026</td>
<td>0.4599</td>
<td>0.1192</td>
<td>0.0193</td>
<td>0.0028</td>
<td>0.0005</td>
</tr>
<tr>
<td>11</td>
<td>NMS</td>
<td>0.8187</td>
<td>0.1820</td>
<td>0.0152</td>
<td>0.0012</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>12</td>
<td>ENs</td>
<td>0.7676</td>
<td>0.1506</td>
<td>0.0111</td>
<td>0.0008</td>
<td>0.0001</td>
<td>0.0426</td>
</tr>
<tr>
<td>13</td>
<td>NMS</td>
<td>0.7676</td>
<td>0.0821</td>
<td>0.0054</td>
<td>0.0004</td>
<td>0.03 e-4</td>
<td>0.03 e-5</td>
</tr>
<tr>
<td>14</td>
<td>ENs</td>
<td>0.6702</td>
<td>0.0824</td>
<td>0.0065</td>
<td>0.0005</td>
<td>0.4 e-4</td>
<td>0.4 e-5</td>
</tr>
</tbody>
</table>

Note: NMS is the nonlinear MDOF system. ENs is the equivalent nonlinear system.
evaluation and calibration of design parameters based on re­
quakes. Comparison of 50-year global and local limit-state
achieved by using two empirical response-modification factors
is satisfactory predictions.

for local (interstory drift) and global (structural drift) responses
global yielding behavior of the system. The equivalence is
probabilities of SMRSF buildings of one to 12 stories at two
sites in Southern California indicate that the method gives sat­
SUMMARY AND CONCLUSIONS

A method of ENS is presented for risk and reliability eval­
uation of MDOF structures in the nonlinear inelastic range. It
is an extension of current approximate methods of equivalent
SDOF systems by considering the first two modes and the
SDOF systems by considering the first two modes and the
these systems; the same is true for irregular structures.

ever, that only regular SMRSFs are considered in this study.
The methodology may be extended to other structural systems
(e.g., braced frames and dual systems). Additional regression
analyses for the equivalent nonlinear systems are necessary for
these systems; the same is true for irregular structures.

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edged.

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FIG. 10. Comparison of Failure Probabilities of ENS and Non­
linear MDOF Systems at Imperial Valley Site: (a) Global Limit
State; (b) Local Limit State

TABLE 5. Local Limit-State Probability (Imperial Valley Site)

<table>
<thead>
<tr>
<th>Structure number</th>
<th>System</th>
<th>0.5% of height</th>
<th>1.0% of height</th>
<th>1.5% of height</th>
<th>2.0% of height</th>
<th>2.5% of height</th>
<th>3.0% of height</th>
</tr>
</thead>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>2</td>
<td>NMS</td>
<td>0.8245</td>
<td>0.6632</td>
<td>0.3532</td>
<td>0.1728</td>
<td>0.0792</td>
<td>0.0330</td>
</tr>
<tr>
<td>2</td>
<td>ENS</td>
<td>0.8257</td>
<td>0.7000</td>
<td>0.3798</td>
<td>0.1761</td>
<td>0.0759</td>
<td>0.0313</td>
</tr>
<tr>
<td>3</td>
<td>NMS</td>
<td>0.8264</td>
<td>0.8237</td>
<td>0.4890</td>
<td>0.2195</td>
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</table>

Note: NMS is the nonlinear MDOF system. ENS is the equivalent nonlinear system.

FIG. 11. Histogram of Deviation of Local Drift Limit-State
Probabilities of ENS from Those of Nonlinear MDOF System

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- $C_0$, $C_1$, $C_2$ = coefficient of two-stage regression analysis;
- $d_i$ = interstory drift at $i$th story;
- $G_s$ = maximum displacement at top of ENS;
- $G_m$ = maximum displacement at top of MDOF system;
- $G_0$ = global drift limit divided by building height;
- $H$ = building height;
- $h_i$ = height of $i$th story;
- $h_i'$ = height of $i$th story;
- $L_o$ = local (interstory) drift limit divided by story height;
- $P_i$ = participation factor of $i$th mode;
- $P_G$ = probability of global drift limit being exceeded;
- $P_L$ = probability of local drift limit being exceeded;
- $R_G$ = global response modification factor = $G_s/G_m$;
- $R_L$ = local response modification factor;
- $u_e$ = maximum roof displacement;
- $u_y$ = global yield displacement;
- $Y$ = global yield displacement from static pushover analysis;
- $y_i'$ = yielding displacement of $i$th story;
- $\gamma$ = ratio of first two modal participation factors to sum of all modes; and
- $\mu$ = global ductility factor.